RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.SC. SECOND SEMESTER (January – June) 2013

Mid-Semester Examination, March 2013

Date : 04/03/2013

MATHEMATICS (Honours)

Time : 11 am – 1 pm

Paper : II

Full Marks : 50

[Use Separate Answer Books for each group]

Group – A

1. Answer **any four** questions : [4×3]

a) Find arg z where $z = 1 + i\tan\frac{3\pi}{5}$

b) If z is a variable complex number such that the ratio $\frac{z-i}{z+1}$ is purely imaginary then show that the point z lies on a circle in the complex plane.

c) Solve : $x^4 + 2x^2 + 4 = 0$

d) Find all values of z such that $\exp\,\bar{z} = 1 + i$.

e) Use De Moivre's theorem to prove that $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$

f) Find log z where $z = 1 + i\tan\theta, \pi/2 < \theta < \pi$

2. Answer **any two** questions : [2×4]

a) Let $\{f(n)\}_{n\in\mathbb{N}}$ be a monotone decreasing sequence of positive real numbers and 'a' be a positive integer greater than 1. Prove that the series $\sum_{n=1}^{\infty} f(n)$ and $\sum_{n=1}^{\infty} a^n f(a^n)$ converge or diverge together.

b) Prove that the series $\frac{a}{b} + \frac{a(a+c)}{b(b+c)} + \frac{a(a+c)(a+2c)}{b(b+c)(b+2c)} + \ldots$, a, b, c > 0 is convergent if b > a+c and divergent if $b \le a + c$.

c) Test the convergence of the following series —

i) $\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \frac{1}{3\cdot4\cdot5} + \ldots$

ii) $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \ldots$ [2+2]

3. Answer **any one** question : [1×5]

a) Let $\{u_n\}_{n\in\mathbb{N}}$ be a monotone decreasing sequence of positive real numbers such that $\lim_{n\to\infty} u_n = 0$. Prove that $\sum_{n=1}^{\infty} (-1)^{n+1} u_n$ is convergent.

b) Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ converges to log 2, but the rearranged series $1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \ldots$ converges to $\frac{3}{2}\log 2$.

Group – B

4. Answer **any two** questions : [2×5]

a) If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$ show that $A^2 - 2A + I_2 = 0$. Hence find A^{50}.

b) Expand by Laplace's method to prove that—

$$\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = (a^2 + b^2 + c^2 + d^2)^2$$

c) A is a non-singular matrix such that the sum of the elements in each row is K. Prove that the sum of the elements in each row of A^{-1} is $\frac{1}{K}$.

5. Answer **any three** : [3×5]

a) Prove that every extreme point of the convex set of all feasible solutions of the system Ax = b, $x \geq 0$ corresponds to a B.F.S.

b) State fundamental theorem of L.P.P If x_1, x_2 be real, show that the set given by $X = \{(x_1, x_2) \mid 9x_1^2 + 4x_2^2 \leq 36\}$ is convex set. [2+3]

c) A factory is engaged in manufacturing two products A and B which involve lathe work, grinding and assembling. The cutting, grinding and assembling times required for one unit of A are 2, 1 and 1 hours respectively and for one unit of B are 3, 1 and 3 hours respectively. The profits on each unit of A and B are Rs. 2·00 and Rs. 3·00 respectively. Assuming that 300 hours of lathe time, 300 hours of grinding time and 240 hours of assembling time, are available, pose a linear programming problem in terms of maximizing the profit on the items manufactured and solve this problem graphically.

d) Find the B.F. solutions of the equations :

$2x_1 + 3x_2 - x_3 + 4x_4 = 8$

$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$

$x_1, x_2, x_3, x_4 \geq 0$

e) Use Charnes M-method to solve the L.P.P.

Minimize $Z = 4x_1 + 3x_2$

subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$x_1, x_2 \geq 0$